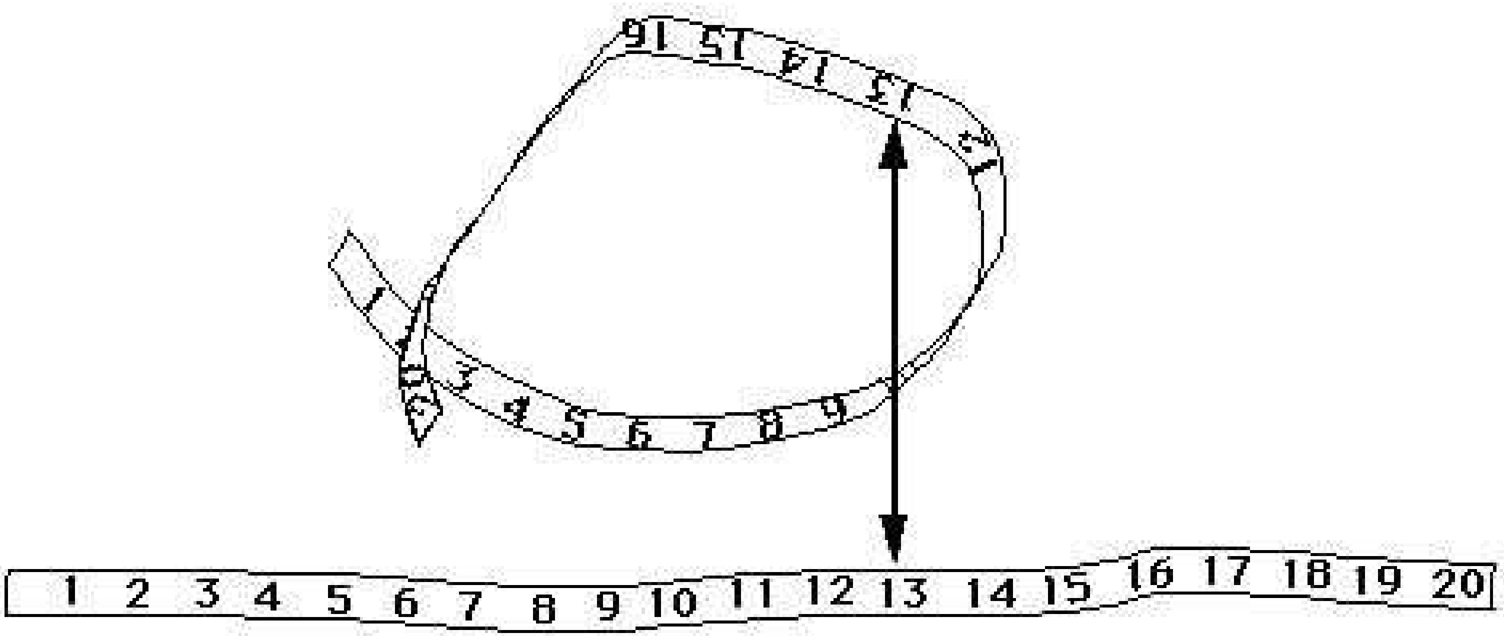
THEOREM OF THE DAY

Brouwer's Fixed Point Theorem *Let B^n denote the n-dimensional closed unit ball and let $f : B^n \longrightarrow B^n$ be a continuous function. Then f has a fixed point: for some $x \in B^n$, $f(x) = x$.*



In one dimension, B^n is just the interval, $[0, 1]$, of the real line consisting of all real numbers from 0 to 1. Equivalently, we can take two tape measures (like the cloth ones used in dress-making) and twist one up. When placed next to the other, untwisted one we will find a fixed point where the numbers are still aligned. In two dimensions B^n is the solid disk of diameter one and Brouwer's fixed point theorem is sometimes illustrated by saying if you place a crumpled up map of the world on top of an uncrumpled copy then some location in the crumpled world will lie directly above the same location in the uncrumpled world.

L.E.J. Brouwer was a leading figure in the early 20th century movement to make mathematics *constructive*: to avoid, for example, proofs by contradiction which eliminate the possibility of something being false without explaining why it must be true. There are many important non-constructive proofs of existence and Brouwer's 1910 Fixed Point Theorem is, ironically, one of them: it tells you there is a fixed point but gives you no way of finding it. The theorem is so important in topology, and has so many applications in areas such as economics, that Brouwer had to acknowledge its canonical place in modern mathematics. He steadfastly refused, however, to lecture on this work.

Web link: cepa.newschool.edu/het/essays/math/fixedpoint.htm

Further reading: *Five Golden Rules: Great Theories of 20th Century Mathematics and Why They Matter* by John L Casti, Wiley, 1997.