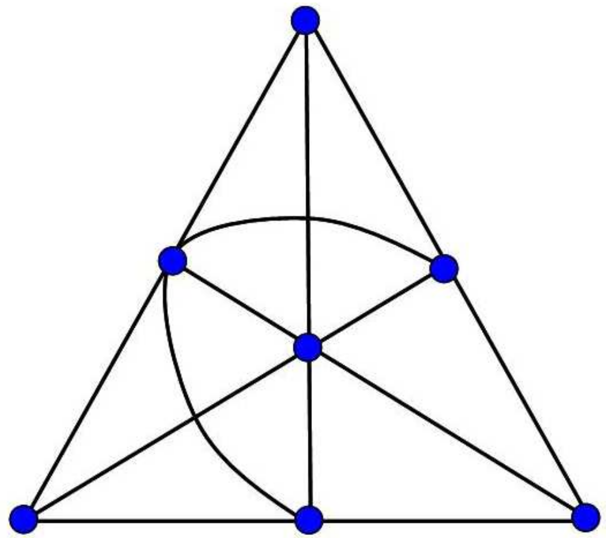
THEOREM OF THE DAY

Theorem (Bruck-Ryser-Chowla) *If a projective plane of order n exists, with $n \equiv 1$ or $2 \pmod 4$ then $n = x^2 + y^2$ for some integers x and y.*



The Fano plane — a projective plane of order 2. A finite projective plane of order n has n^2+n+1 points and n^2+n+1 lines, arranged so that

1. every line contains $n+1$ points, and every point is on $n+1$ lines,
2. any two distinct lines intersect at exactly one point, and any two distinct points lie on exactly one line.

One of the most famous problems in combinatorics is to confirm or disprove that finite projective planes only exist for prime power order. The Bruck-Ryser-Chowla theorem (proved by Bruck and Ryser in 1949 and generalised a year later by Ryser and Chowla) eliminates many possible counterexamples, starting with $6 = 4+2$, $14 = 3\times 4+2$, $21 = 5\times 4+1$ and $22 = 5 \times 4 + 2$, which are not sums of squares. The numbers $10 = 1^2 + 3^2$, $12 \equiv 0 \pmod 4$, $15 \equiv 3 \pmod 4$ and $18 = 3^2 + 3^2$ escape the net. The non-existence of a projective plane of order 10 (having 111 points) was proved by Clement Lam and colleagues in 1991 after a Herculean combination of mathematical reasoning and computer search. The cases 12, 15 and 18 do not seem to be within reach at present.

Web link: www.cecm.sfu.ca/organics/papers/lam/paper/html/node2.html

Further reading: *Designs, Graphs, Codes and their Links* by P. J. Cameron and J. H. van Lint, Cambridge University Press, Cambridge, 1991.