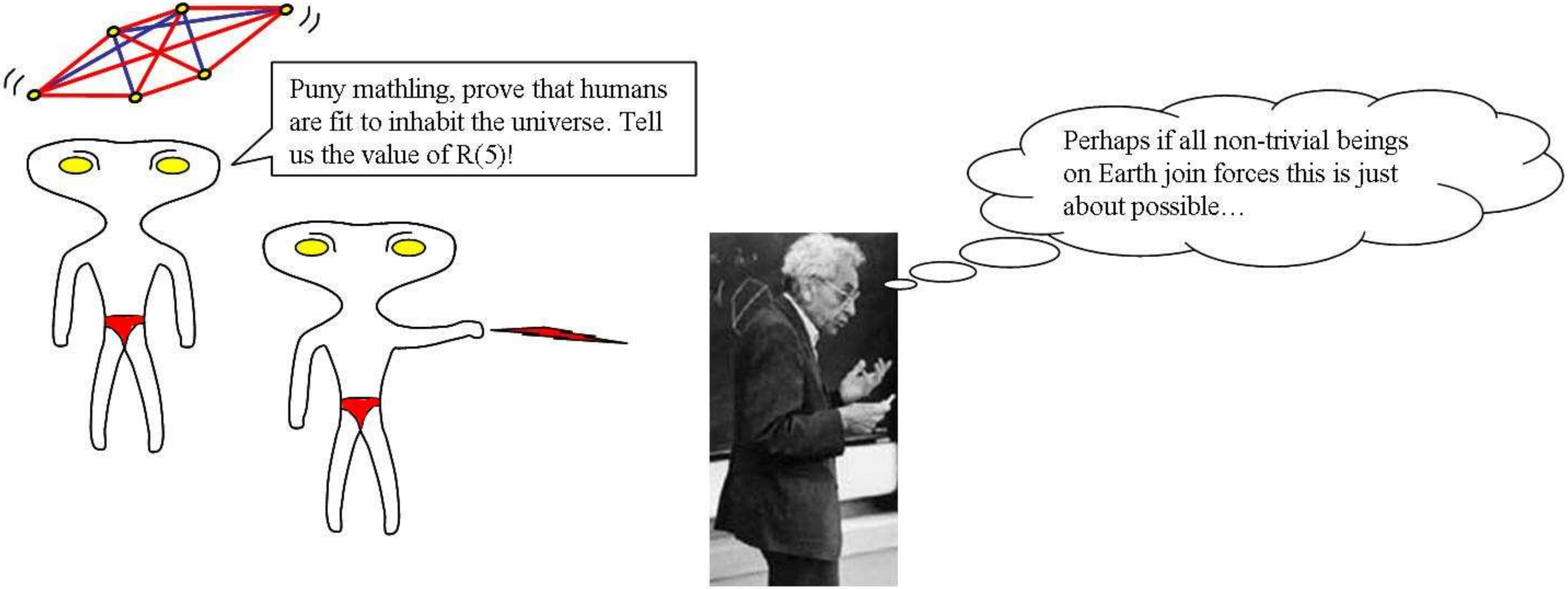
THEOREM OF THE DAY

Ramsey's Theorem *For each positive integer n, there is a least positive integer R(n) such that, if the edges of the complete graph on R(n) nodes are coloured red and blue, then it contains a complete subgraph with n nodes whose edges are either all red or all blue.*



In the picture, the aliens' spaceship is a complete graph K_6 missing two diagonal edges. Since the value of $R(3)$ is known to be 6, if these are added in either colour, a single-coloured K_3 (i.e. a red or blue triangle) will be created.

Only small values of $R(n)$ are known: for n =1, 2, 3, 4, the values of $R(n)$ are 1, 2, 6 and 18, respectively. The great Paul Erdős reputedly said "If aliens threatened to destroy the Earth unless we gave them the value of $R(5)$ then it would be worthwhile to put all our efforts into finding this number. If they demanded the value of $R(6)$ then we should put all our efforts into killing the aliens." So far only crude upper and lower bounds for $R(n)$ are known: $43 \leq R(5) \leq 49$ and $102 \leq R(6) \leq 165$.

Before he died tragically at the age of 26 of liver failure, F.P. Ramsey made contributions in philosophy, economics and mathematics. His theorem was published in 1930, the year of his death, in a paper entitled *On a Problem of Formal Logic*. It has since given birth to a whole branch of combinatorics called Ramsey Theory.

Web link: `cranmer.home.cern.ch/cranmer/html/ramsey_theory.html`. Plus: David Cariolaro offers £500 in a Ramsey Theory Contest (closes June '09) to find a non-computer proof that $R(3,7) \leq 23$, where $R(m,n)$ is the smallest N such that any red-blue K_N must contain a red K_m or a blue K_n.

Further reading: *A Walk Through Combinatorics, 2nd Edition* by Miklós Bóna, World Scientific Publishing, 2006.