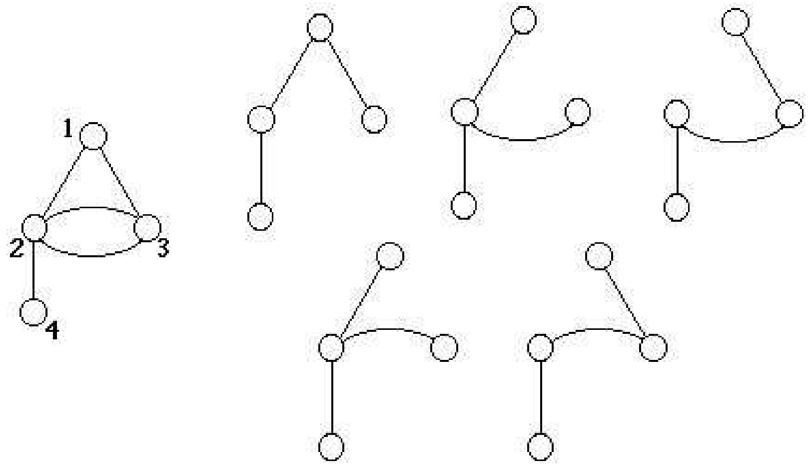
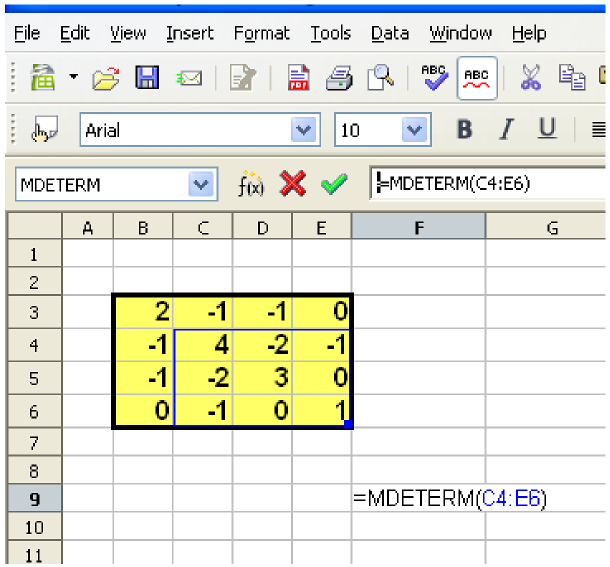
THEOREM OF THE DAY

The Matrix Tree Theorem (Kirchoff) *Let G be a graph with n vertices and let $L(G)$ be the $n \times n$ matrix whose entry in row i and column j is defined to be*

$$\begin{array}{ll} -(\textit{the number of edges joining vertex } i \textit{ to vertex } j) & \textit{if } i \neq j, \quad \textit{and} \\ \textit{the number of edges incident with vertex } i & \textit{if } i = j. \end{array}$$

Then the number of spanning trees of G is given by $\det L(G)(1|1)$, *where $L(G)(1|1)$ is the matrix obtained by deleting the 1st row and 1st column of $L(G)$.*





The graph on the far left has 5 edges on $n = 4$ vertices. Its spanning trees, in the centre, are those subsets of $n - 1 = 3$ edges which contain no cyclic paths. The *determinant* function, det, yields a single figure from a square matrix or table. It is available in standard spreadsheet applications as the '=MDETERM' function. As shown here in the OpenOffice Calc package, the calculation will produce the answer 5, since there are exactly 5 spanning trees for the given graph. In fact, any row of $L(G)$, not just the first, and any column, may be deleted in the statement of the theorem without changing the absolute value of the result.

This calculation was first devised by Gustav Kirchoff in 1847 as a way of obtaining values of current flow in electrical networks. Matrices were first emerging as a powerful mathematical tool about the same time.

Web link: www.math.ku.edu/~jmartin/mc2004/graph1.pdf

Further reading: *Introduction to Graph Theory, 4th Ed.* by Robin Wilson, Longman, 1996.